Mathematics functions grade 12 pdf

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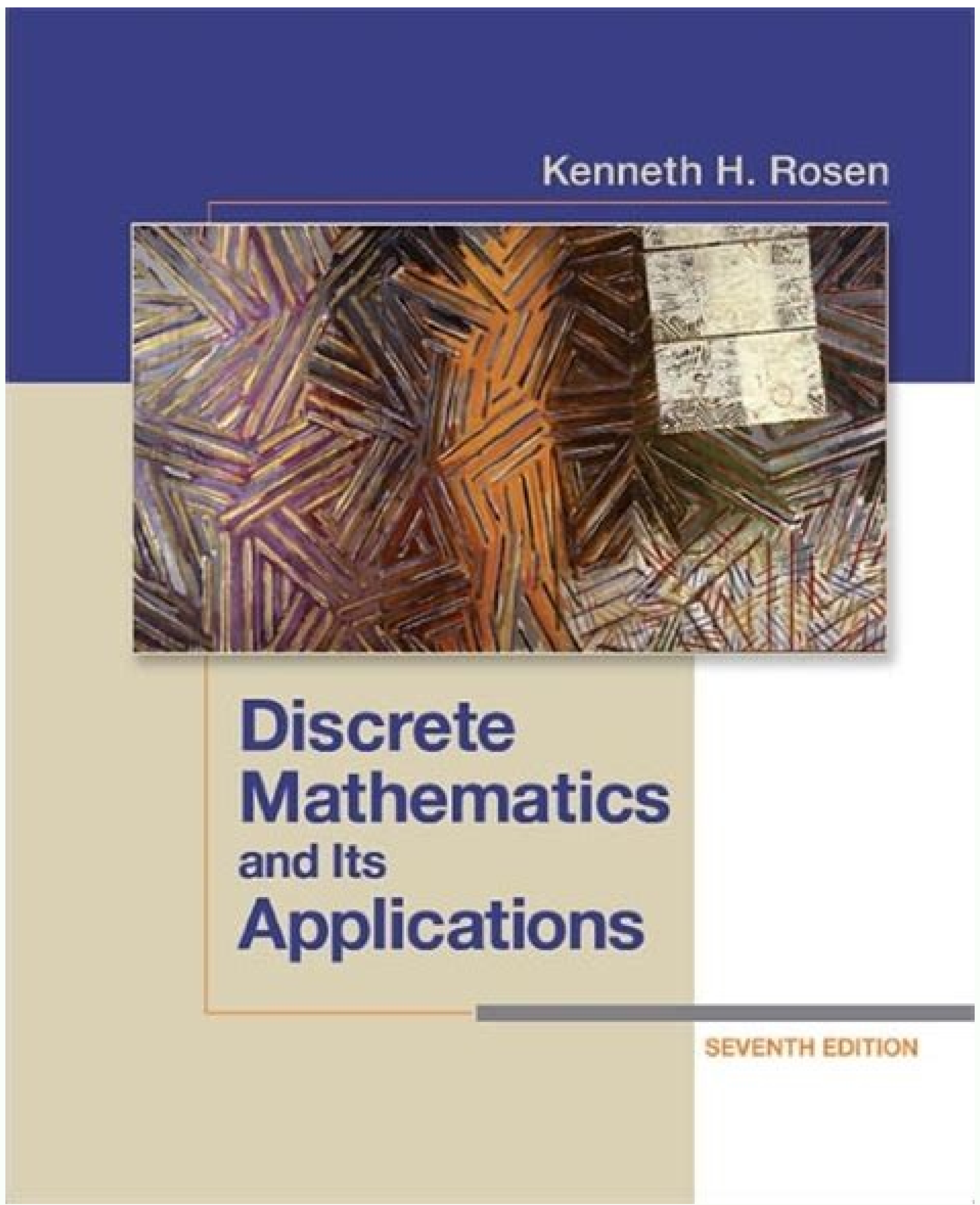
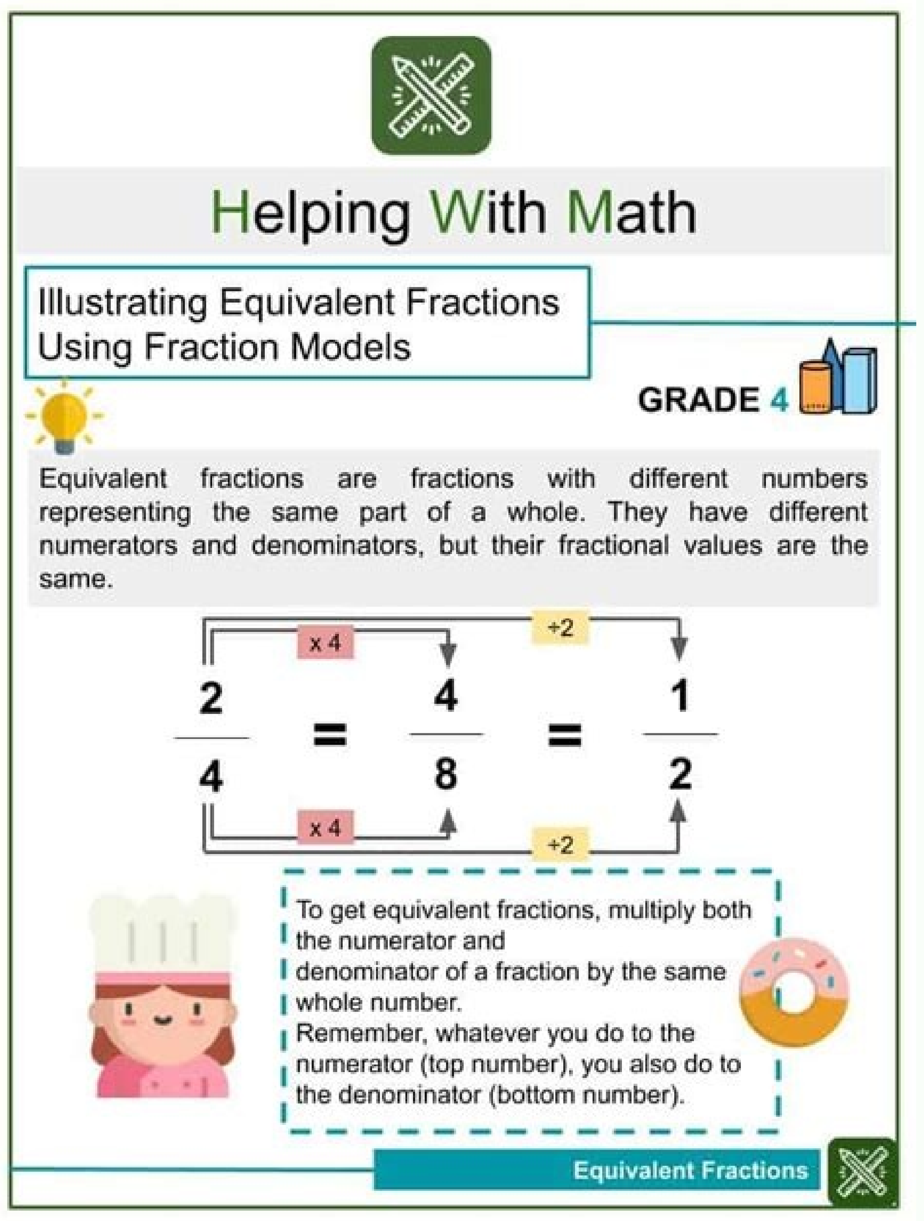
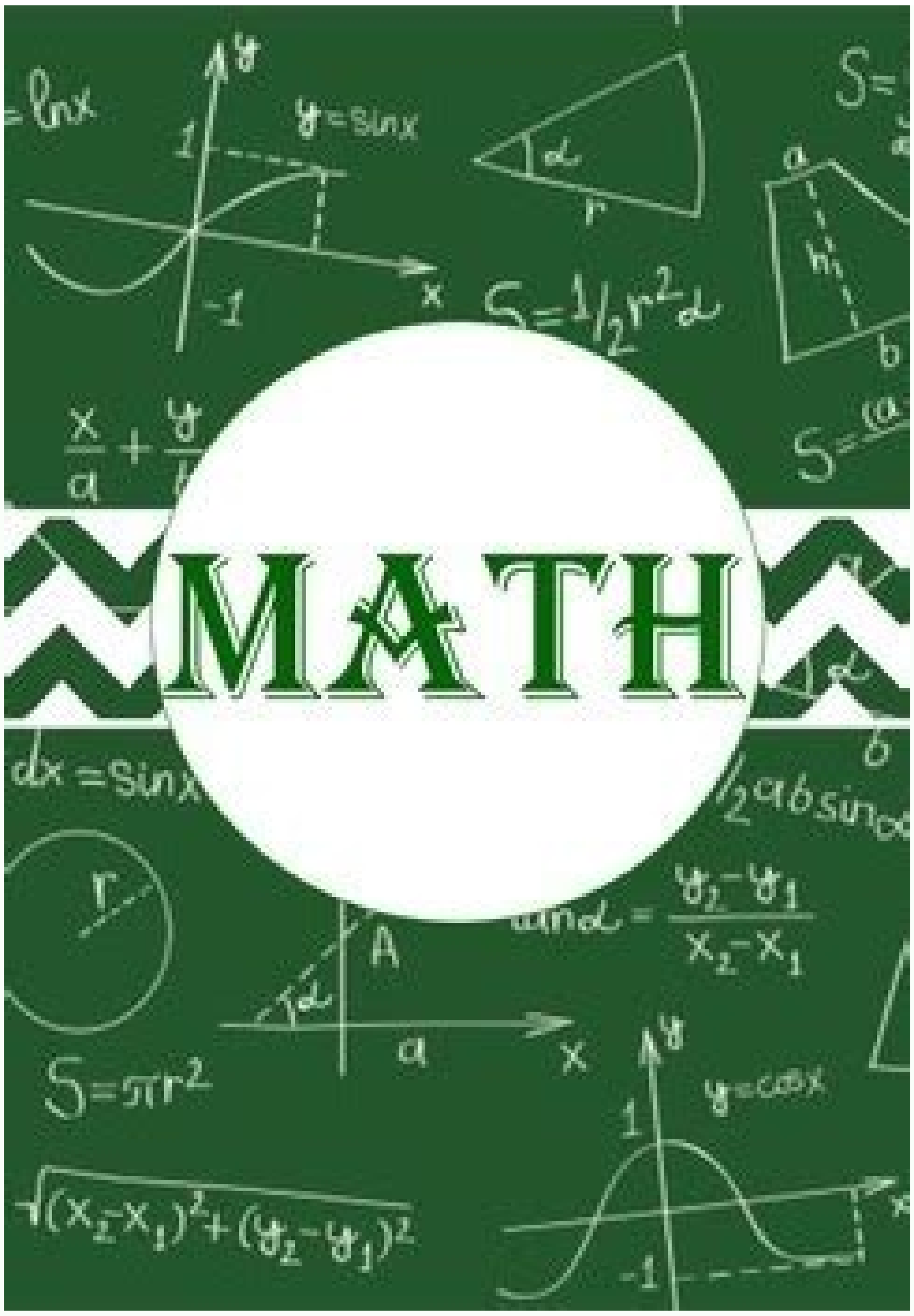
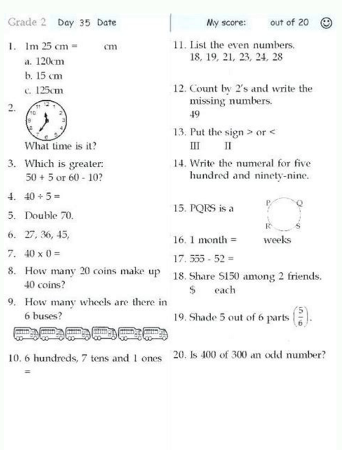
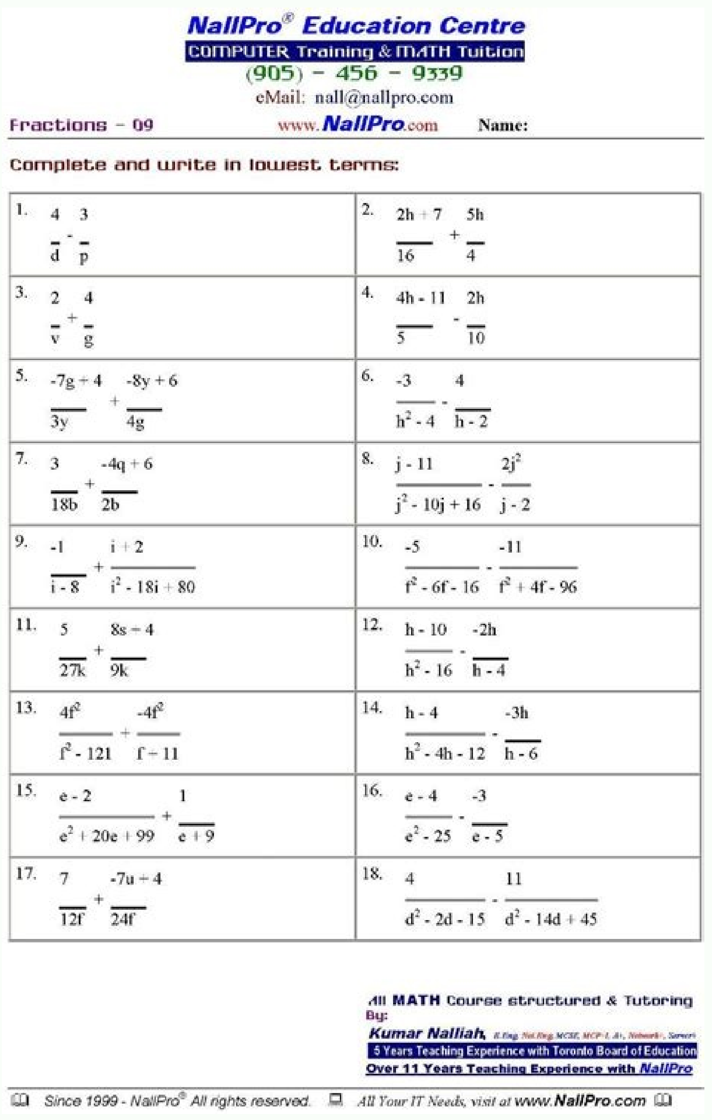

Kenneth H. Rosen
Discrete Mathematics and Its Applications
SEVENTH EDITION


Helping With Math
Illustrating Equivalent Fractions Using Fraction Models
GRADE 4
Equivalent fractions are fractions with different numbers representing the same part of a whole. They have different numerators and denominators, but their fractional values are the same.
x 4
÷2
2/4 = 4/8 = 1/2
x 4
÷2
To get equivalent fractions, multiply both the numerator and denominator of a fraction by the same whole number.
Remember, whatever you do to the numerator (top number), you also do to the denominator (bottom number).
Equivalent Fractions



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Complete and write in lowest terms:

1. $\frac{4}{d} - \frac{3}{p}$	2. $\frac{2h+7}{16} + \frac{5h}{4}$
3. $\frac{2}{v} + \frac{4}{g}$	4. $\frac{4h-11}{5} - \frac{2h}{10}$
5. $\frac{-7g+4}{3y} + \frac{-8y+6}{4g}$	6. $\frac{-3}{h^2-4} - \frac{4}{h-2}$
7. $\frac{3}{18b} + \frac{-4q+6}{2b}$	8. $\frac{j-11}{j^2-10j+16} - \frac{2j^2}{j-2}$
9. $\frac{-1}{i-8} + \frac{i+2}{i^2-18i+80}$	10. $\frac{-5}{f^2-6f-16} - \frac{-11}{f^2+4f-96}$
11. $\frac{5}{27k} + \frac{8s-4}{9k}$	12. $\frac{h-10}{h^2-16} - \frac{-2h}{h-4}$
13. $\frac{4f^2}{f^2-121} + \frac{-4f^2}{f-11}$	14. $\frac{h-4}{h^2-4h-12} - \frac{-3h}{h-6}$
15. $\frac{e-2}{e^2+20e+99} + \frac{1}{e+9}$	16. $\frac{e-4}{e^2-25} - \frac{-3}{e-5}$
17. $\frac{7}{12f} + \frac{-7u+4}{24f}$	18. $\frac{4}{d^2-2d-15} - \frac{11}{d^2-14d+45}$

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Learners must be encouraged to check whether or not the inverse is a function. It is very important that learners understand that \(f^{-1}(x)\) notation can only be used if the inverse is a function. Learners must not confuse the inverse function \(f^{-1}\) and the reciprocal \(\frac{1}{f(x)}\). Encourage learners to state restrictions, particularly for quadratic functions. Learners must understand that \(y = \sqrt{-x}\) has real solutions for \(x < 0\). Exercises on parabolic functions with horizontal and vertical shifts have been included for enrichment only and are clearly marked. The logarithmic function is introduced as the inverse of the exponential function. Learners need to understand that the logarithmic function allows us to rewrite an exponential expression with the exponent as the subject of the formula. It is very important that learners can change from exponential form to logarithmic form and vice versa. This skill is also important for finding the period of an investment or loan in the Finance chapter. Learners should be encouraged to use the definition and change of base to solve problems. Manipulation involving the logarithmic laws is not examinable. Learners should be encouraged to be familiar with the LOG function on their calculator and also to use their calculator to check answers. Enrichment content is not examinable and is clearly marked. In previous grades we learned about the characteristics of linear, quadratic, hyperbolic and exponential functions. In this chapter we will demonstrate the ability to work with various types of functions and relations including inverses. In particular, we will look at the graphs of the inverses of: Linear functions: \(y=mx+c\) or \(y = ax + q\) Quadratic functions: \(y=a{x}^{2}\) Exponential functions: \(y=b^{x} \quad (b > 0, b e 1)\) Draw a graph of \(2y + x - 8 = 0\) and determine the significant characteristics of this linear function. Write the equation in standard form \(y = mx + c\) \begin{align*} 2y + x - 8 &= 0 \\ 2y &= - x + 8 \\ \therefore y &= -\frac{1}{2}x + 4 \\ \therefore m &= -\frac{1}{2} \\ \text{And } c &= 4 \end{align*} To draw the straight line graph we can use the gradient-intercept method: \begin{align*} y-\text{intercept}: \enspace & (0;4) \\ m &= -\frac{1}{2} \end{align*} Alternative method: we can also determine and plot the \(x\)- and \(y\)-intercepts as follows: For the \(y\)-intercept, let \(x = 0\): \begin{align*} y &= -\frac{1}{2}(0) + 4 \\ \therefore y &= 0 + 4 \\ &= 4 \end{align*} This gives the point \((0;4)\). For the \(x\)-intercept, let \(y = 0\): \begin{align*} 0 &= -\frac{1}{2}x + 4 \\ \frac{1}{2}x &= 4 \\ \therefore x &= 8 \end{align*} This gives the point \((8;0)\). Gradient: \(-\frac{1}{2}\) Intercepts: \((0;4) \text{ and } (8;0)\) Domain: \(\{x: x \in \mathbb{R} \}\) Range: \(\{y: y \in \mathbb{R} \}\) Decreasing function: as \(x\) increases, \(y\) decreases. Write the quadratic function \(2y - x^{2} + 4 = 0\) in standard form. Draw a graph of the function and state the significant characteristics. \begin{align*} 2y - x^{2} + 4 &= 0 \\ 2y &= x^{2} - 4 \\ y &= \frac{1}{2}x^{2} - 2 \end{align*} Therefore, we see that: \[a = \frac{1}{2}; \qquad b = 0; \qquad c = -2\] For the \(y\)-intercept, let \(x = 0\): \begin{align*} y &= \frac{1}{2}(0)^{2} - 2 \\ \therefore y &= 0 - 2 \\ &= -2 \end{align*} This gives the point \((0;-2)\). For the \(x\)-intercept, let \(y = 0\): \begin{align*} 0 &= \frac{1}{2}x^{2} - 2 \\ 0 &= x^{2} - 4 \\ 0 &= (x - 2)(x + 2) \\ \therefore x = -2 &\text{ or } x = 2 \end{align*} This gives the points \((-2;0)\) and \((2;0)\). Shape: \(a > 0\), therefore the graph is a "smile". Intercepts: \((-2;0), (2;0)\text{ and } (0;-2)\) Turning point: \((0;-2)\) Axes of symmetry: \(x = -\frac{b}{2a} = - \frac{0}{2 \left(\frac{1}{2} \right)} = 0\) Domain: \(\{ x: x \in \mathbb{R} \}\) Range: \(\{ y: y \geq -2, y \in \mathbb{R} \}\) The function is decreasing for \(x < 0\) and increasing for \(x > 0\). Draw the graphs of \(f(x) = 2^{x}\) and \(g(x) = \left(\frac{1}{2} \right)^{x}\) on the same set of axes and compare the two functions. Consider the function: \(f(x) = 2^{x}\) \begin{align*} \text{If } y = 0: \quad 2^{x} &= 0 \\ \text{But } 2^{x} &e 0 \\ \therefore \text{ no solution} & \\ \text{If } x = 0: \quad 2^{0} &= 1 \\ \text{This gives the point } & (0;1). \end{align*} Asymptotes: \(f(x) = 2^{x}\) has a horizontal asymptote, the line \(y = 0\), which is the \(x\)-axis. \(x\) \(-2\) \(-1\) \(0\) \(1\) \(2\) \(f(x)\) \(\frac{1}{4}\) \(\frac{1}{2}\) \(1\) \(2\) \(4\) Consider the function: \(g(x) = \left(\frac{1}{2} \right)^{x}\) \begin{align*} \text{If } y = 0: \quad \left(\frac{1}{2} \right)^{x} &= 0 \\ \text{But } \left(\frac{1}{2} \right)^{x} &e 0 \\ \therefore \text{ no solution}& \\ \text{If } x = 0: \quad \left(\frac{1}{2} \right)^{0} &= 1 \\ \text{This gives the point } & (0;1). \end{align*} Asymptotes: \(g(x) = \left(\frac{1}{2} \right)^{x}\) also has a horizontal asymptote at \(y = 0\). \(x\) \(-2\) \(-1\) \(0\) \(1\) \(2\) \(g(x)\) \(4\) \(2\) \(1\) \(\frac{1}{2}\) \(\frac{1}{4}\) Symmetry: \(f\) and \(g\) are symmetrical about the \(y\)-axis. Domain of \(f\) and \(g\): \(\{ x: x \in \mathbb{R} \}\) Range of \(f\) and \(g\): \(\{ y: y > 0, y \in \mathbb{R} \}\) The function \(g\) decreases as \(x\) increases and function \(f\) increases as \(x\) increases. The two graphs intersect at the point \((0;1)\). Textbook Exercise 2.1 \(f(x) = 3x^{2}\) and \(g(x) = - x^{2}\) For \(f(x)\): \begin{align*} \text{Intercept: } & (0;0) \\ \text{Turning point: } & (0;0) \\ \text{Axes of symmetry: } & x = 0 \\ \text{Domain: } & \{x: x \in \mathbb{R} \} \\ \text{Range: } & \{y: y \geq 0, y \in \mathbb{R} \} \\ \text{Minimum value: } & y = 0 \end{align*} For \(g(x)\): \begin{align*} \text{Intercept: } & (0;0) \\ \text{Turning point: } & (0;0) \\ \text{Axes of symmetry: } & x = 0 \\ \text{Domain: } & \{x: x \in \mathbb{R} \} \\ \text{Range: } & \{y: y \leq 0, y \in \mathbb{R} \} \\ \text{Maximum value: } & y = 0 \end{align*} \(j(x) = - \frac{1}{5}x^{2}\) and \(k(x) = - 5x^{2}\) For \(j(x)\): \begin{align*} \text{Intercepts: } & (0;0) \enspace (0;0) \\ \text{Turning point: } & (0;0) \\ \text{Axes of symmetry: } & x = 0 \\ \text{Domain: } & \{x: x \in \mathbb{R} \} \\ \text{Range: } & \{y: y \leq 0, y \in \mathbb{R} \} \\ \text{Maximum value: } & y = 0 \end{align*} For \(k(x)\): \begin{align*} \text{Intercepts: } & (0;0) \enspace (0;0) \\ \text{Turning point: } & (0;0) \\ \text{Axes of symmetry: } & x = 0 \\ \text{Domain: } & \{x: x \in \mathbb{R} \} \\ \text{Range: } & \{y: y \leq 0, y \in \mathbb{R} \} \\ \text{Maximum value: } & y = 0 \end{align*} \(h(x) = 2x^{2} + 4\) and \(l(x) = - 2x^{2} - 4\) For \(h(x)\): \begin{align*} \text{Intercept: } & (0;4) \\ \text{Turning point: } & (0;4) \\ \text{Axes of symmetry: } & x = 0 \\ \text{Domain: } & \{x: x \in \mathbb{R} \} \\ \text{Range: } & \{y: y \geq 4, y \in \mathbb{R} \} \\ \text{Minimum value: } & y = 4 \end{align*} For \(l(x)\): \begin{align*} \text{Intercept: } & (0;-4) \\ \text{Turning point: } & (0;-4) \\ \text{Axes of symmetry: } & x = 0 \\ \text{Domain: } & \{x: x \in \mathbb{R} \} \\ \text{Range: } & \{y: y \leq -4, y \in \mathbb{R} \} \\ \text{Maximum value: } & y = -4 \end{align*} Given \(f(x) = -3x - 6\) and \(g(x) = mx +c\). Determine the values of \(m\) and \(c\) if \(g \parallel f\) and \(g\) passes through the point \((1;2)\). Sketch both functions on the same system of axes. \begin{align*} g(x) &= mx +c \\ m &= -3 \\ g(x) &= -3x +c \\ \text{Substitute } (1;2) \quad 2 &= -3(1) + c \\ \therefore c &= 5 \\ \therefore g(x) &= -3x + 5 \\ \text{Intercepts for } g: \quad & (\frac{5}{3};0);(0;5) \\ & \\ \text{Intercepts for } f: \quad & (-2;0);(0;-6) \end{align*} Given \(m: \frac{x}{2} - \frac{y}{3} = 1\) and \(n: - \frac{y}{3} = 1\). Determine the \(x\)- and \(y\)-intercepts and sketch both graphs on the same system of axes. For \(m(x)\): \begin{align*} \frac{x}{2} - \frac{y}{3} & = 1 \\ \text{Let } x = 0: \quad - \frac{y}{3} & = 1 \\ y & = -3 \\ \text{Let } y = 0: \quad \frac{x}{2} & = 1 \\ x & = 2 \\ \text{Intercepts: } & (2;0);(0;-3) \end{align*} For \(n(x)\): \begin{align*} - \frac{y}{3} & = 1 \\ \therefore y & = -3 \\ \text{Intercepts: } & (0;-3) \end{align*} Write the linear function in standard form: \begin{align*} \frac{x}{2} - \frac{y}{3} & = 1 \\ \frac{3x}{2} - y & = 3 \\ \frac{3x}{2} - 3 & = y \\ \therefore y &= \frac{3x}{2} - 3 \end{align*} Sketch \(p, q \text{ and } r\) on the same system of axes. For each of the functions, determine the intercepts, asymptotes, domain and range. For \(p(x)\): \begin{align*} \text{Intercept: } & (0;1) \\ \text{Asymptote: } & y = 0 \\ \text{Domain: } & \{x: x \in \mathbb{R} \} \\ \text{Range: } & \{y: y > 0 \} \end{align*} For \(q(x)\): \begin{align*} \text{Intercept: } & (0;1) \\ \text{Asymptote: } & y = 0 \\ \text{Domain: } & \{x: x \in \mathbb{R} \} \\ \text{Range: } & \{y: y > 0 \} \end{align*} For \(r(x)\): \begin{align*} \text{Intercept: } & (0;-1) \\ \text{Asymptote: } & y = 0 \\ \text{Domain: } & \{x: x \in \mathbb{R} \} \\ \text{Range: } & \{y: y < 0 \} \end{align*} If you are given a set of x-values, you can work out the set of y-values or answers that came from using a given rule on each x-value.So there is a relationship between the x-values and the y-values that is described by the rule.The x-values are the input values and the y-values are the output values. In this flow diagram, the rule is y = 2x – 1So for every x-value, we multiply it by 2 and subtract 1 to find the corresponding y-value.The input values or x-values are the elements of the domain of this set and the output values or y-values are the elements of the range of this set.We can plot these values on the Cartesian plane.If we extend the domain so that x∈, we get the graph for y = 2x – 1.Look at the graph. For every x-value on this graph, there is only one y-value. If a rule or a formula produces only one y-value for each x-value, then we have a function.A function is a relationship between x and y, where for every x-value there is only one y-value.One way to decide whether or not a graph represents a function is to use the vertical line test.If any line drawn parallel to the y-axis cuts the graph only once, then the graph represents a function.e.g 1Graph A and Graph B are functions.Graph C is not a function because the vertical cuts the graph twice. So for an x-value on the graph, there are two y-values. 4.2 Function notation We use function notation f(x) to show that each y-value is a function of an x-value.We can also use other letters too, such as g(x), h(x), etc.So y = 2x – 1 can be written as f(x) = 2x – 1.The value of f(x) for any x-value can be worked out by substitution:For example, at x = –3 we can find f(–3) = 2(–3) – 1 = - 7So the point (–3; –7) lies on the graph of f(x) = 2x – 1 Activity 1 If h(x) = (½)x determine the value of h(–4). (3) If the function g(x) = – x2 –3x, find g(x + h) (2) If f(x) = 4x + 1, determine the value of:3.1 f(x + a)3.2 f(x) + a3.3 af(x) (3) If g(x) = 2x2, determine the value of:4.1 g(–x)4.2 –g(x) (2)[10] Solutions h(x) = (½)x ∴ h(–4) = (½)-4 (2–1)–4 = 24 = 16So when x = –4, y = 16 and the point (–4; 16) lies on the graph of the function 3h. (3) g(x) = –x2 – 3x∴g(x + h) = –(x + h)2 –3(x + h) wherever there is an x, replace it with (x + h)= –(x2 + 2xh + h2) – 3x – 3h= –x2 – 2xh – h2 – 3x – 3hThis means that when x = x + h, y = –x2 – 2xh – h2 – 3x – 3h (2) 3.1 f(x) = 4x + 1 f(x + a) = 4(x + a)+1= 4x + 4a + 13.2 f(x) = 4x + 1 f(x) + a = 4x + 1 + a3.3 f(x) = 4x + 1af(x) = a(4x + 1)= 4ax + a (3) 4.1 g(x) = 2x2g(–x) = 2(–x)2= 2x24.2 g(x) = 2x2–g(x) = –2x2 (2)[10] In each example, there is only one possible y-value for each x-value, so f(x); h(x) and g(x) are functions. 4.3 The basic functions, formulas and graphs Important terms to remember:Domain: the set of possible x-valuesRange: the set of possible y-valuesAxis of symmetry: an imaginary line that divides a graph into two mirror images of each other.Maximum: the highest possible y-value of a function.Minimum: the lowest possible y-value of a function.Asymptote: an imaginary line that a graph approaches but never touches.Turning point: The point at which a graph reaches its maximum or minimum value and changes direction.4.3.1 The linear function (straight line)Linear functions have the form f(x) = ax + q where a represents the gradient of a straight-line graph and q represents the y-intercept when x = 0.The graph of y is a straight line with a = 1 and q = 0Domain: x ∈ RRange: y ∈ RAlso note the shape of the following linear functionsSKETCHING THE LINEAR FUNCTIONTo sketch the linear function using the dual intercept method. Determine the x-intercept (let y = 0) Determine the y-intercept (let x = 0) Plot these two points and draw a straight line through them. DETERMINING THE EQUATION OF A LINEAR FUNCTIONTo determine the equation of the linear function follow the following steps: Determine the gradient of the function. Substitute the value of the gradient into the general formula for the linear function. Solve for q. Write the equation in the form f(x) = ax + q e.g 2 Solutions a = y2 – y1 x2–

x1= –1 – 0 1 – 2 a = 1∴ y = 1x + c0 = 1(2) + cc = –2∴ f(x) = x – 2 a = y2 –y1 x2– x1= 2 – 0 - 1 – 0 a = –2 ∴ y = -2x + c 0 = –2(0) + c 3 c = 0 ∴ f(x) = x – 2x [5] 4.3.2 The quadratic functions (parabola)A quadratic function is a parabola and can be represented with a general formula y = ax2 + bx + c or y = a(x + p)2 + q[PROPERTIES OF A PARABOLA] SKETCHING THE QUADRATIC FUNCTIONTo sketch any quadratic function, follow the following steps: Write down the y-intercept (let x = 0) To calculate the x-intercepts, Write the equation in the form ax2 + bx + c = 0 Factorise the left hand side of the equation. Use the fact that if (x – p)(x – q) = 0, then x = p or x = q, to calculate the x-intercepts. Determine the axis of symmetry. Substitute the x-value of the axis of symmetry into the original equation of the function to calculate the co-ordinates of the turning point. Plot the points and then draw the function using free hand. e.g. 3Sketch the graph of f(x) = x2 – 5x – 6 y-interceptf(0) = –6Therefore the co-ordinates of the y-intercept are (0; –6) 3 x-interceptx2 – 5x – 6 = 0 3(x – 6)(x + 1) = 0 3x = 6 or x = –1 3(6; 0) and (–1; 0) Axis of symmetryx = – b 2a= –(–5) 2(1)= 5 2 Turning point Sketch Graph Determining the equation of a quadratic function Given the x-intercept and one point Given the turning point and one point Use the formula: y = a(x – x1)(x – x2). Substitute the values of the x-intercepts. Substitute the given point which is not the x-intercept. Solve for a. Write the equation in the formf (x) = ax2 + bx + c. Use the formula: y = a(x + p)2 + q. Substitute the co-ordinates of the turning point (p; q). Substitute the given point. Solve for a. Write the equation in the formy = a(x +p)2 + q or f (x) = ax2 + bx + cdepending on the instruction in the question. Given the co-ordinates of three points on the parabola Use the formula: y = ax2 + bx + c. One of the given point is the y-intercept, therefore c is given, so substitute its value. Substitute the co-ordinates of the other two points into y = ax2 + bx + c. Solve the two equations simultaneously for a and b. Nature of the roots and the quadratic function Nature of roots Quadratic function Real rootsΔ > 0 Equal rootsΔ = 0 Non-real rootsΔ > 0 Activity 2The sketch represents the graph of the parabola given by f(x) = 2 – x – x2.Points A, B and C are the intercepts on the axes and D is the turning point of the graph.1.1 Determine the co-ordinates of A, B and C. (4)1.2 Determine the co-ordinates of the turning point D. (3)1.3 Write down the equation of the axes of symmetry of f(x– 5). (1)1.4 Determine the values of x for which – f(x) > 0. (2)[10] Solutions1.1B(0; 2)2 – x – x2 = 0 x2 + x – 2 = 0(x – 1)(x + 2) = 0 x = 1 or x = –2 A(–2; 0) and C(1; 0) 3 (4) 1.2 x = –b 2a= – (–1) 2(–1)= -½f (-½) = 2 – (-½) – (-½)2=9/4 = 2¼D (-½;9/4)1.3 x = 9 or x = 4½ (1)1.4 x ≤ –2 or x ≥ 1 (2)[10] Activity 3The sketch represents the graph of the parabola given by f(x) = ax2 + bx + c and the straight line defined by g(x) = mx + cPoints A, B ,C and D are the intercepts on the axes. E is the point of intersection of the two graphs.2.1 Write down the co-ordinates of point D if D is the image of B after B has been translated two units to the right. (1)2.2 Determine the equation of g. (3)2.3 Determine the equation of the function f in the form f(x) = ax2 + bx + c. (4)2.4 Determine the coordinates of E. (4)2.5 Write down the values of x for which f(x) ≥ g(x). (2)[14] Solutions2.1 D(5; 0) 3 (1)2.2 g(x) = mx + 30 = m(5) + 3 or mg = 3 – 0 = – 3 0 - 5 5m = – 3 /5 g(x) = – 3 /5 x + 3 (3)2.3 f(x) = a(x + 1)(x – 3) 3 = a(0 + 1)(0 – 3) a = 1 f(x) = –(x + 1)(x – 3)f(x) = –x2 + 2x + 3 (4) 2.4 – 3 /5x + 3 = –x2 + 2x + 3 x2 – 13/5 x = 02.5 0 ≤ x ≤ 13/5 (2)[14] 4.3.3 The hyperbolic functionHyperbola of the form y = a or xy = a where a ≠ 0; x ≠ 0; y ≠ 0.PropertiesShape Domain : x ∈ R; x ≠ 0 Range: y ∈ R; y ≠ 0 The horizontal asymptote is the x-axis The vertical asymptote is the y-axis If a < 0, the graph lies in the 2nd and 4th quadrant If a > 0 , the graph lies in the 1st and 3rd quadrant The lines of symmetry are: y = x and y = –x. SKETCHING THE HYPERBOLA OF THE FORM:y = a or xy = a x The graph does not cut the x-axis and the y-axis (asymptotes) Use the table and consider both the negative and positive x-values a determine two quadrants where the graph will be drawn Activity 4 1. Sketch the graph of y = 1/x by plotting points.Describe the main features of the graph. (4)Solutiona = 1a > 0, the graph lies in the 1st and 3rd quadrant -3 - 2 - 1 - ½ 0 ½ 1 2 3 -1/3 - ½ -1 -2 undefined 2 1 - ½ 1/3 Domain: x ∈ R; x ≠ 0 Range: y ∈ R; y ≠ 0 Asymptotes: x = 0 and y = 0 Lines of symmetry y = x and y = –x (4) 2. Sketch the graph of y = –4/x by plotting the points. Describe the main features of the graphs. (4)Solutiona = –4a < 0, the graph lies in the 2nd and 4th quadrant -4 -2 -1 0 1 2 4 1 2 4 undefined -4 -2 -1 Domain: x ∈ R; x ≠ 0 Range: y ∈ R; y ≠ 0 Asymptotes: x = 0 and y = 0 Lines of symmetry y = x and y = –x (4) [8] 4.3.4 The hyperbolaHyperbola of the form y = a/x + q is the translation of the graph of y = a/x vertically by q units.The Horizontal asymptote (x–axis) will also shift q units vertically (up or down). Activity 5 1. Consider the function y = 1/x – 21.1 Determine : the equations of the asymptotes the coordinates of the x–intercepts 1.2 Sketch the graph1.3 Write down: the domain and range the lines of symmetryy = x + c and y = –x + c(10) Solutions1.1 The horizontal asymptote is y = –2 since the graph moved 2 units down and the vertical asymptote is x = 0 denominator cannot equal to zero. For x – intercepts let y = 00 = 1 – 2 0 = 1/x – 2x (multiplying by LCD which is x)2x = 1 x = ½(½;0) 2. Consider the function f(x) = –4/x+ 12.1 Determine: the equations of the asymptotes the coordinates of the x–intercepts 2.2 Sketch the graph2.3 Write down the domain and range2.4 If the graph of f is reflected by the line having the equation y = –x + c, the new graph coincides with the graph of f(x).Determine the value of c.(9) Solutions2.1 The horizontal asymptote is y = 1 since the graph moved1 units up and the vertical asymptote is x = 0 denominator cannot equal to zero. For x–intercepts let y = 00 = -4/x + 10 = –4 + x (m3ultiplying by LCD which is xx = 4(4; 0) 1.2 x -4 -2 -1 0 1 2 4 y -2¼ -2½ -3 undefined -1 -1½ -13/4 1.3 Domain: x ∈ R; x ≠ 0 Range: y ∈ R; y ≠ 2 y = x and y = –xtranslation 2 units down thereforey = x – 2 and y = –x – 2 ∴ c = –2Or substitute (0; 2) point of intersection of the two asymptotes iny = x + c or y = –x + cAnd calculate the value of c[10] Compare this graph with the one in activity 4 (a) 2.2 x -4 -2 -1 0 1 2 4 y 2 2 5 undefined -3 -1 0 2.3 Domain: x ∈ R; y ≠ 0 Range: y ∈ R; y ≠ 1 2.4 The asymptotes arex = 0 and y = 1y = –x + c1 = –(0) + c1 = clines are y = –x + 1 and y = 3x + 1[9]Compare this graph with the one in activity 4 (b) 4.3.5 Hyperbola of the formy = a + q where a ≠ 0, x ≠ 0, y ≠ 0 x + p ShapeThe dotted lines are the asymptotes Domain: x ∈ R; x ≠ –p. Range: y ∈ R; y ≠ q The horizontal asymptote is y = q The vertical asymptote is x + p = 0 ∴x = –p The lines of symmetry are y = x + c and y = x + c e.g. 4Consider g(x) = 8 –3 has the horizontal asymptote at y = –3 and x – 2 ≠ 0 ∴x ≠ 2 because if x = 2 the denominator of the expression 8 would be 8 = 8 x + 2 x - 2 2 - 2 0which is undefined because the denominator is zero.Thus the graph is undefined for x – 2 = 0 ∴ x = 2 is the vertical asymptoteThe graph y = 8/x shift 2 units to the right and 3 units down to form the graph g(x) = 8 - 3 x - 2SKETCHING THE HYPERBOLA OF THE FORMy = a + q x + p Write down the asymptotes Draw the asymptotes on the set of axes as dotted lines Use a to determine the two quadrants where the graph will be drawn Determine the x – intercept(s) let y = 0 Determine the y – intercept(s) let x = 0 Plot the points and then draw the graph using free hand Activity 6 Consider the function f(x) = 2 + 1 x - 3 Write down the equations of the asymptotes of f (2) Calculate the coordinates of the x and y-intercepts of f (4) Write the domain and range (2) Sketch the graph of f clearly showing ALL asymptotes and intercepts with the axes. (3) Consider the function f(x) = 3 – 2 x - 1 Write down the equation of the asymptotes. (2) Calculate the coordinates of the intercepts of the graph of f with the axes. (3) Sketch the graph of f clearly showing the intercepts with the axes and the asymptotes. (3) Write down the range of y = –f(x). (1) Describe, in words, the transformation of f to g if g(x) = -3 – 2 (2) x + 1[22] Solution x = 3 and y = 1 (2) f(x) = 2 + 1 x - 3y – intercept y = 2 + 1 = 1 0 - 3 3 3(0;1/3)x – intercept 0 = 2 + 1 x - 30 = 2 + 1(x – 3)0 = 2 + x – 3x = 1 ∴(1; 0) (4) Domain: x ∈ R ; x ≠ 3 Range: y ∈ R ; y ≠ 1 (2) a > 0 intercepts asymptotes shape (3)[11] Solution2. 3 x = –1 y = –2 (2) y – intercepty = 3 – 2 = –5 0 - 1(0; –5)x – intercept 0 = 3 –2 x - 12 = 3 x - 12(x – 1) = 32x – 2 = 32x = 5x = 5 2(5/2 ; 0) (3)interceptsasymptotesshape (3) a > 0 f(x) = 3 – 2 x - 1– f(x) = – (3 – 2) x - 1– f(x) = – 3 + 2 x - 1Range: y ∈ R; y ≠ 2 (1) g(x) = - 3 – 2 x + 1g(x) = 3 – 2 -x -1Since x is negative this is the reflectionof f about the y-axis (2)[11] In the graph 1 (d) the points (4; 3), x = 4 was chosen because it has x-coordinate greater thanx = 3 the vertical asymptote. The point (2; –1), was chosen because has x-coordinate x = 2 isless than x = 3 the vertical asymptote. These points can also be used to help determining in which quadrants the graph must be drawn. The points (2; 1) and (–2; –3) on graph 2 (iii) were chosen similarly.Activity 7The diagram below represents the graph of f(x) = a + q. T(5; 3) is a point on f. x + p4.1 Determine the values of a, p and q (4)4.2 If the graph of f is reflected across the line having the equation y = –x + c, the new graph coincides with the graph of y = f(x).Determine the value of c. (3)[7] Solutions4.1 p = 4 and q = 2 using the asymptotesSubstitute T(5; 3) into y = a + 2 x - 43 = a + 2 5 - 43 = a + 2 a = 1 (4)4.2 Substitute (4; 2) 3into y = –x + c2 = –(4) + c ∴ c = 6 (3)[7] Activity 8Sketched below are the graphs of f(x) = (x + p)2 + q and g(x) = a + c A(2½;0) is a point on the graph of f. P is the turning point of f. The asymptotes of g are x + b represented by the dotted lines. The graph of g passes through the origin5.1 Determine the equation of g. (4)5.2 Determine the coordinates of P, the turning point of f. (4)5.3 Write down the equation of the asymptotes of g(x – 1). (2)5.4 Write down the equation of h, if h is the image of f reflected about the x–axis. (1)[11] Solutions5.1 Using the asymptotes 3b = 1 and c = 2Substitute (0; 0) into y = a + 2 x - 10 = a + 2 ⇒ 0 = –a + 2 ∴ a = 2 0 - 1y = 2 + 2 (4) x - 15.2 Axis of symmetry p = 1f(x) = (x – 1)2 + q(5/2;0)0 = (5/2 - 1)2 + q0 = 9/4 + qq = - 9/4 ∴ P (1; -9/4) (4)5.3 g(x) = 2 + 2 x - 1g(x – 1) = 2 + 2 substitute x with (x – 1) (x - 1) - 1g(x – 1) = 2 + 2 x - 2x = 2 and y = 2 (2)5.4 f(x) = (x – 1)2 – 9/4Reflection about the x – axis y changes the sign– y = (x – 1)2 - 9/4y =– [(x – 1)2 – 9/4]y = –(x – 1)2 + 9/4 (1)[11] 4.3.6 The exponential functionAn exponential function can be represented with a general formula y = abx+p + q; b > 0Shape and properties of an exponential function y = bx; b > 1 y = bx; 0 < b > 1 The graph passes through the point (0; 1). Domain: x ∈ R Range: y > 0 but for y + bx + q, the range will be at y > q. The graph is smooth, continuous and an increasing function. Asymptote is at y = 0 but for y = bx + q, the horizontal asymptote will be at y = q. The graph passes through the point (0; 1). Domain: x ∈ R Range: y > 0 but for y = bx + q, the range will be at y > q. The graph is smooth, continuous and a decreasing function. Asymptote is at y = 0 but for y = bx + q, the horizontal asymptote will be at y = q. NOTE: The two functions are a reflection of each other about the y-axis. e.g. 5Given: f(x) = 2x1.1 Draw the graph of f(x) = 2x, show at least three points on the sketch.1.2 Draw, on the same system of axes the graph of f –1, the inverse of f.1.3 Write down the equation of f –1 in the form y = ... Solutions1.1 Start by drawing the table: Then plot the graph using the points1.2 The sketch of f –1 is obtained by interchanging the x and y co-ordinates of f.1.3 y = 2xx = 2yy = log2 x [2] e.g. 6The sketch represents the graph given by f(x) = ax.2.1 Write down the coordinates of point A. (1)2.2 How can we tell that 0 < a < 1? (1)2.3 Determine a if B is the point (3;1/27) . (2)2.4 Determine the equation of the graph obtained if f is reflected about the y–axis. (2)2.5 What are the coordinates of the point of intersection of the two graphs? (1)[7] Solutions2.1 A(0; 1) 2.2 Because the graph is a decreasing function. 2.3 f(x) = ax1/27 = a3 (3 –1)3 = a3a = 1/32.4 f(x) = (1/3)xy = (1/3)x becomes y = (1/3)-x∴y = (3 –1) –xy = 3x 2.5 (0; 1) [7] Activity 9The curve of an exponential function is given by f(x) = kx and cuts the y–axis at A (0; 1) while B (2:9/4) lies on the curve.Determine1.1 the equation of the function f. (3)1.2 the equation of the asymptote of h if h(x) = –f(x). (2)1.3 the range of h. (1)1.4 The equation of the function g of which the curve is the reflection of the curve of f in the line y = x. (2) Solutions1.1 f(x) = kx9/4 = a2 (3/2)2 = a2 a = 3/2 ∴f(x) = (3/2)x (3)1.2 y = 0 (2)1.3 y ≤ 0 (1)1.4 g(x) = log 3/2 x (2)[8] 4.4 Inverse functions The inverse of a function takes the y-values (range) of the function to the corresponding x-values (domain) and vice versa. Therefore the x and y values are interchanged. The function is reflected along the line y = x to form the inverse. The notation for the inverse of a function is f –1. e.g. 7Given f(x) = 2x + 6. Determine f –1(x) Sketch the graphs of f(x), f –1 (x) and y = x on the same set of axis Solutions In order to find the inverse of a function, there are two steps:STEP 1: Swap the x and yy = 2x + 6 becomes x = 2y + 6 We then rewrite the equation to make y the subject of the formula.Therefore,STEP 2: make y the subject of the formulax = 2y + 6x – 6 = 2y So y = ½ x – 3 We can say that the inverse function f –1(x) = ½x - 3 Every point on the function has the same coordinates as the corresponding point on the inverse function, except that they are swapped around. Example: (–3; 0) on the function is reflected to become (0; –3) on the inverse function. Any point (a; b) on the function becomes the point (b; a) on the inverse. To find the equation of an inverse function algebraically, we interchange x and y and then solve for y. To draw the graph of the inverse function, we reflect the original graph about the liney = x, the axis of symmetry of the two graphs. e.g. 8 Sketch f(x) = 2x2 Determine the inverse of f(x) Sketch f –1(x) and y = x on the same axes as f(x) Solution1. b) y = 2x2x = 2y2 y = ± √x/2 This is not a function. Check it with a vertical line test There are two y-values for one x-value. Not all inverses of functions are also functions. Some inverses of functions are relations. If an inverse is not a function, then we can restrict the domain of the function in order for the inverse to be a function. To make the inverse a function, we need to choose a set of x-values in the function and work only with those. We call this ‘restricting the domain’. A one to one function has an inverse that is a functionExample: y = 3x + 4 is a one to one function. For every x value there is one and only one y valueThe inverse of is a function. A many to one function has an inverse that is not a function. However, we can restrict the domain of the function to make its inverse a function.Example: y = 2x2 is a many to one function. For two or many x values there is one y value.(if x = 2, then y = 8.If x = –2, then y = 8). Therefore, its inversey = ± √x/2 is not a function. To check for a function, draw a vertical line. If any vertical line cuts the graph in only one place, the graph is a function.If any vertical line cuts the graph in more than one place, then the graph is not a function. To check for a one-to-one function, draw a horizontal line. If any horizontal line cuts the graph in only one place, the graph is a one-to-one function. If any horizontal line cuts the graph in more than one place, then the graph is a many-to-one function. [5] Activity 10 If f(x) = –3x2 ,write down the equation for the inverse function in the form y = ………………… (2) Determine the domain and range of f(x) and f –1 (x) (4) Determine the points of intersection of f(x) and f –1 (x) (4) If g(x) = 3x + 2, find g –1(x) (2) Sketch g, g –1 and the line y = x on the same set of axes. (3)[15] Solutions For f(x) = – 3x2.f –1 (x): x = –3y2-x/3 = y2y = ± √-x/3 (2) f(x) f–1(x) Domain x ∈ R x ≥ 0 Range y ≥ 0 y ∈ R (4) To determine the points of intersection, we equate the two equations.The line y = x, the axis of symmetry of f(x) and f –1(x), can also be used to determine the points of intersection of f(x) and f –1(x).y = x and f(x) = − 3x2∴ x = −3x2∴ 3x2 + x = 0 ∴ x(3x + 1) = 0 ∴ x = 0 or x =-1/3Substitute x = 0 in y = x ∴ y = 0 ∴ (0; 0) Substitute x =-1/3 in y = x ∴ y = -1/3 ∴ (-1/3 ; 1/3) (4) g(x) = 3x + 2 For g −1 (x), x = 3y + 2x – 2 = 3yy = x − 2 3y = x − 2 3 3(4)[15] Given: g (x) = – x2 where x < 0 and y < 0 Write down the inverse of g, g–1 in the form h(x) = ………. (3) Sketch the graphs of g, h and y = x on the same set of axis. (4) Solutions y = –x2x = –y2– x = y2 ± √–x = y2– √–x = y where x < 0 and y < 0∴ h(x) = – √–x (3) For g correct shape 3and the intercept 3For h correct shape 3and the intercept 3 (4)[7] 4.5 The logarithmic function y = logxa is a logarithmic function with a = log number, x=log base y = logxa Reads “y is equal to log a base x” The logarithmic function is only defined if a > 0, a ≠ 1 and x > 0 An exponential equation can be written as a logarithmic equation and vice versa e.g. 9Write each of the following exponential equations as logarithmic equations: Solutions 26 = 64∴ 6 = log2 64 53 = 125∴ 3 = log5 125 e.g. 10Given: f(x) = 2x Determine f –1 in the form y = …… Sketch the graphs of f(x), f –1 (x) and y = x on the same set of axes. Write the domain and range of f (x) and f –1 (x) The inverse of the exponential function y = ax is x = ay In order to make y the subject of the formula, x = ay, we use the log function. y = log ax is the inverse of y = ax. Solutions The inverse of the exponential function y = 2x is x = 2y which can be written as y = log2 x. To plot the graph, use a table of values:First make a table for y = x -2 -1 0 1 2 3 y = 2x ¼ ½ 1 2 4 8 Make a table for y = log2 x x 1 2 4 8 y = log2x -2 -1 0 1 2 3 [3] Let’s compare the two graphs on the Cartesian plane.The graph of y = log2 x is a reflection about the y = x axis of the exponential graph of y = 2x. Activity 11The graph of h(x) = ax is sketched below. A (–1; ½) is a point on the graph of h. Solutions h(0) = a0 = 1. Any base raised to the power of 0 is 1. (2) h(x) = ax and A(–1; ½) so a –1 = ½a –1 = 2 –1 so a = 2 and y = 2x (2) Interchange x and y, so x = 2y and y = log2 x (1) (2) x > 0.5 (1)[8] What you need to be able to do: Understand the concept of the inverse of a function and find the equations of the inverses. The line y = x is the line of symmetry of the function and the inverse of the function The logarithmic function and the exponential function are inverse functions of each other. If the inverse is not a function, restrict the domain of a function in order to make the inverse a function Identify axes of symmetry for parabolas and hyperbolas Sketch the graphs of different functions using their characteristics e.g. asymptotes, x- and y –intercepts and turning points Determine the functions equations from a graph Solve problems involving two or more graphs Understand the concept of the inverse of a function and the equation of the inverses The line y = x is the line of symmetry of the function and the inverse of the function The logarithmic functions and the exponential function are inverse function of each other If the inverse is not a function, restrict the domain of a function in order to make the inverse a function.

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